International Junior Math Olympiad

GRADE 10

Time Allowed: 90 minutes

Name:

Country:

INSTRUCTIONS

1. Please DO NOT OPEN the contest booklet until told to do so.
2. There are 30 questions.

   Section A: Questions 1 to 10 score 2 points each, no points are deducted for unanswered question and 1 point is deducted for wrong answer.

   Section B: Questions 11 to 20 score 3 points each, no points are deducted for unanswered question and 1 point is deducted for wrong answer.

   Section C: Question 21 to 30 score 5 points each, no points are deducted for unanswered or wrong answer.
3. Shade your answers neatly using a 2B pencil in the Answer Entry Sheet.
4. No one may help any student in any way during the contest.
5. No electronic devices capable of storing and displaying visual information is allowed during the exam. Strictly NO CALCULATORS are allowed into the exam.
6. No exam papers and written notes can be taken out by any contestant.

SECTION A – 10 questions

Question 1

ABC is an equilateral triangle. A circle with radius 1 is tangent to the line AB at the point B and to the line AC at point C. What is the side length of ABC?

A. $\frac{\sqrt{3}}{2}+1$

B. $\sqrt{3}$

C. $\frac{\sqrt{3}}{2}$

D. $2\sqrt{3}$

E. 2

Question 2

You throw three regular six-sided dice. What is the probability that you will get one odd number and two even numbers?

A. $\frac{1}{4}$

B. $\frac{3}{8}$

C. $\frac{4}{27}$

D. $\frac{1}{2}$

E. $\frac{1}{3}$

Question 3

If $m = 2(3)(4)(5) \dots (31)(32)$, which statement about m is true?

A. $m < 2^{40}$

B. $2^{40} < m < \ 2^{70}$

C. $2^{70} < m < 2^{100}$

D. $2^{100} < m < 2^{130}$

E. $2^{130} < m$

Question 4

Real numbers *a* and *b* satisfy the equations $3^a = 81^{b+2}$ and $125^b = 5^{a-3}$. What is *ab*?

A. -60

B. -17

C. 9

D. 12

E. 60

Question 5

What are the last two digits of 2017^{2017}?

A. 77

B. 81

C. 93

D. 37

E. 57

Question 6

Students from Mrs. Hein's class are standing in a circle. They are evenly spaced and consecutively numbered starting with 1. The student with number 3 is standing directly across from the student with number 17. How many students are there in Ms. Hein's class?

A. 28

B. 29

C. 30

D. 31

E. 32

Question 7

A singles tournament had six players. Each player played every other player only once, with no ties. If Helen won 4 games, Ines won 3 games, Janet won 2 games, Kendra won 2 games and Lara won 2 games, how many games did Monica win?

A. 0

B. 1

C. 2

D. 3

E. 4

Question 8

Points A, B, C, D, and E are on a line such that AB = 3, BC = 6, CD = 8, and DE = 4. What is the smallest possible value of AE?

A. 0

B. 1

C. 2

D. 3

E. 5

Question 9

Which one of the following numbers is equal to $\frac{2017^4-2016^4}{2017^2+2016^2}$?

A. 2016

B. 4031

C. 4033

D. $2 \times (2017^2 - 2016^2)$

E. 2016×2017

Question 10

There are 20 students in a class. If one new boy joins the class, there will be twice as many boys as girls in the class. What is the product of the number of boys and the number of girls in the class?

A. 75

B. 84

C. 91

D. 96

E. 100

Section B – 10 questions

Question 11

Per, Ragnar, and Lars live in the same neighbourhood. They have found out that the straight line distance from Per's house to Ragnar's house is 250 m, and from Ragnar's house to Lars' house is 300 m. Which of the following is true about the distance between Per's house and Lars' house?

A. The distance is precisely 550 m.

B. The distance is between 0 m and 550 m.

C. The distance is between 50 m and 550 m.

D. The distance is between 250 m and 300 m.

E. The distance can be any value.

Question 12

Let n be the largest integer such that $n \leq 2017$ and the sum of its digits equals the product of its digits. Find the sum of the digits of n.

A. 2

B. 3

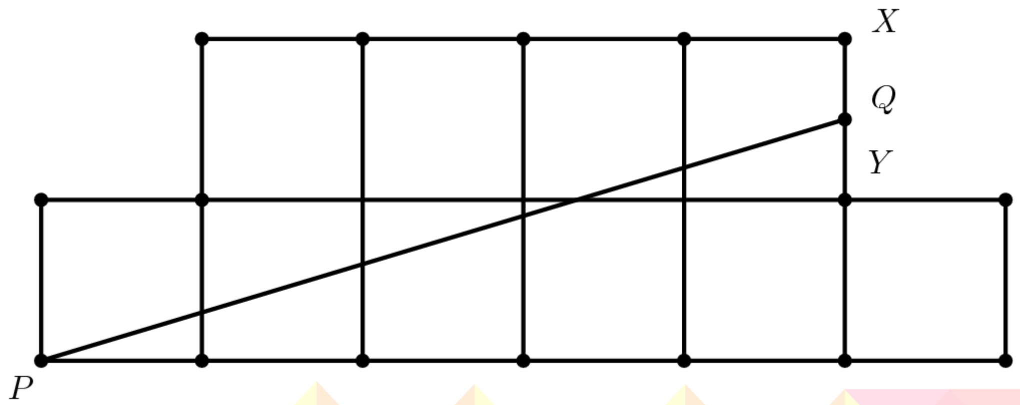
C. 4

D. 6

E. 8

Question 13

The diagram shows an octagon consisting of 10 unit squares. The shapes below *PQ* is a unit square and a triangle with base 5. If PQ divides the area of the octagon into two equal parts, what is the value of $\frac{XQ}{QY}$?



A. $\frac{2}{5}$

B. $\frac{1}{2}$

C. $\frac{3}{5}$

D. $\frac{2}{3}$

E. $\frac{3}{4}$

Question 14

If $a_1 + a_2 = 1, a_2 + a_3 = 2, a_3 + a_4 = 3, a_4 + a_5 = 4, \dots\ a_{50} + a_{51} = 50$ and $a_{51} + a_1 = 51$, what is the sum of $a_1, a_2, a_3, \dots, a_{51}$?

A. 663

B. 1326

C. 1076

D. 538

E. 665

Question 15

Which one of these numbers must be placed in the middle (3rd) if they are to be arranged in increasing or decreasing order?

A. π

B. $\sqrt{12}$

C. $\frac{7}{2}$

D. $\frac{\sqrt{11}+\sqrt{13}}{2}$

E. $\frac{2}{\frac{1}{\sqrt{11}}+\frac{1}{\sqrt{13}}}$

Question 16

The numbers a_1, a_2, a_3 and a_4 are drawn one at a time from the set {0, 1, 2, …, 9}. If these four numbers are drawn with replacement, what is the probability that $a_1a_4 - a_2a_3$ is an even number?

A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. $\frac{3}{8}$

D. $\frac{3}{4}$

E. $\frac{5}{8}$

Question 17

What is $3a^b + 8a^{-3b}$, if $a^b = 2$?

A. 5

B. 7

C. 8

D. 24

E. 70

Question 18

Arne has a box with 100 chips of colours red, white, blue, and black. Each chips has only one colour. Arne told Berit that she (Berit) must pick at least 81 chips from the box to be sure of getting at least one of each colour, if she picks them without looking. After some thought, Berit concluded correctly that the box contains at least N chips of each colour, but at most M of each. What is the smallest possible value of $M - N$?

A. 0

B. 5

C. 20

D. 40

E. 60

Question 19

Emmy is playing with a calculator. She enters an integer, and takes its square root. Then she repeats the process with the integer part (round down) of the answer. After the third process, the integer part equals 1 for the first time. What is the difference between the largest and the smallest number Emmy could have started with?

A. 229

B. 231

C. 239

D. 241

E. 254

Question 20

Peter has three boxes, with ten balls in each. He plays a game in which the goal is to end up with few balls as possible in the boxes. The boxes are each marked with one number from {4,7,10}. It is allowed to remove N balls from the box marked with the number N, put three of them aside, and put the rest in another box. What is the least total number of balls in the boxes can contain in the end?

A. 0

B. 2

C. 3

D. 5

E. 6

Section C – 10 questions

Question 21

Let $p(x) = x^4 + ax^3 + bx^2 + cx + d$ where a, b, c, d be real numbers. It is known that $p(1) = 841, p(2) = 1682$ and $p(3) = 523$. Find $\frac{p(9)+p(-5)-4}{-16}$.

Question 22

Consider the sequence of real numbers (a_n), for n = 1, 2, 3, …,

$a_1 = 2$ and $a_n = (\frac{n+1}{n-1})(a_1 + \cdots + a_{n-1})$ for all n ≥ 2.

Determine the value of $\frac{a_{2017}}{2^{2017}}$.

Question 23

There are eight positive integers in a row. Starting from the third, each is the sum of the two numbers before it. If the eighth number is 2017, what is the largest possible value of the first one?

Question 24

The third, fourth, seventh and last terms of a non-constant (numbers are different) arithmetic progression form a geometric progression. Find the number of terms of this progression.

Question 25

The function $f(x)$ has the following properties

i. $f\left(\frac{x}{x+1}\right) = \frac{1}{2}f(x)$

ii. $f(1-x) = 1 - f(x)$

for all positive real numbers x. Find the value of $S_n = f(1) + f\left(\frac{1}{2}\right) + f\left(\frac{1}{3}\right) + \cdots + f(\frac{1}{n})$.

Question 26

Given that $5r + 4s + 3t + 6u = 100$, where r ≥ s ≥ t ≥ u ≥ 0 are real numbers, find the sum of the maximum and minimum possible values of $r + s + t + u$.

Question 27

There are 10 children in a row. In the beginning, the total number of marbles girls have were equal to the total number of marbles boys have. Then each child gave a marble to every child standing to the right of him (or her). After that, the total number of marbles girls have increased by 25. How many girls are there in the row?

Question 28

Anna and Birger walks at the same constant speed. They start at the same place, facing in the same direction. Birger walked in that direction throughout the time. Anna, however, turned 90 degrees to the right immediately after her first step. Then she turned another 90 degrees to the right immediately after taking two more steps, and yet another 90-degree turn to the right immediately after taking four more steps. She walked this way, doubling the number of steps every time she turns. When Anna was about to turn for the 9th time, both of them stopped Find the integer part of the distance between them when they stopped.

Question 29

The sum of several distinct positive integers is equal to 30. Find the maximum value of their product.

Question 30

Let a, b, x, y be real number such that:

$$a + b = 23;\ ax + by = 79;\ ax^2 + by^2 = 217;\ ax^3 + by^3 = 691$$

What is the value of $ax^4 + by^4$?

END OF PAPER

1	B
2	B
3	A
4	E
5	A
6	A
7	C
8	B
9	C
10	C
11	C
12	E
13	D
14	A
15	D
16	E
17	B
18	C
19	C
20	C
21	5621
22	1009
23	0244
24	0016
25	0002
26	0044
27	0005
28	0323
29	6720
30	1993